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The possibility of measuring spatial velocity correlation functions in a turbulent fluid by means of light scattering†

Abstract. The aim of this letter is to show in which way the conditional probability of light diffused by particles suspended in a turbulent fluid can yield direct information on spatial velocity correlation functions.

The relation between the conditional probability of light scattered by particles suspended in a turbulent fluid and velocity correlation functions has been already considered, both from an experimental (Bourke *et al.* 1969) and a theoretical (Di Porto *et al.* 1969) point of view. It has been shown that the probability density that a photon is registered at time τ by means of a counter with 'quantum efficiency' α, provided one has occurred at time $t = 0$ (Mandel and Wolf 1965), is

$$p_c(\tau) = \alpha \frac{\langle |E(t)|^2 |E(0)|^2 \rangle}{\langle |E(t)|^2 \rangle} \tag{1}$$

where $\boldsymbol{E}$ represents the diffused electric field and the angular brackets stand for ensemble averages. Further, p_c obeys the relation (Di Porto *et al.* 1969)

$$p_c(\tau) = \alpha \langle I \rangle \left[1 + \exp\left\{-k^2 \overline{U_1{}^2} \int_0^\tau \int_0^\tau R_L(t''-t')\,dt'\,dt''\right\} \right.$$
$$\left. \times \left\langle \exp\left\{k^2 \overline{U_1{}^2} \int_0^\tau \int_0^\tau \tilde{R}_L(t''-t'; \boldsymbol{r}_{i,0} - \boldsymbol{r}_{j,0})\,dt'\,dt''\right\} \right\rangle_P \right] \tag{2}$$

where $\langle I \rangle$ is the average intensity scattered at an angle ν and $k = 2k_0 \sin(\nu/2)$, k_0 being the wavenumber of the incident radiation. The right hand side of equation (2) depends on the dynamics of the fluid through the mean square value of velocity fluctuations $\overline{U_1{}^2}$ along an arbitrary direction (if one conside isotrrsopic turbulence), the normalized Lagrangian velocity correlation function $R_L(t''-t')$ (Hinze 1959) and the normalized Lagrangian correlation function $\tilde{R}_L(t''-t'; \boldsymbol{r}_{i,0} - \boldsymbol{r}_{j,0})$ generalized to different fluid elements. The symbol $\langle \ldots \rangle_P$ labels an ensemble averaging operation over the initial positions $\boldsymbol{r}_{i,0}$, $\boldsymbol{r}_{j,0}$ of two different particles. It is worthwhile to remember that the validity of equation (2) rests on the assumption of joint Gaussian probability density distribution for the velocity of turbulence. This hypothesis is somewhat questionable, but it has recently received experimental confirmation for weak turbulent fields (Long and Huang 1970).

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Let us now consider the experiment sketched in figure 1. A monochromatic plane laser beam is diffused by a turbulent liquid doped with small polystyrene balls. The experimental set-up is such that sufficiently narrow diaphragms enable the collection of light scattered at right angles by two nearly equal scattering volumes

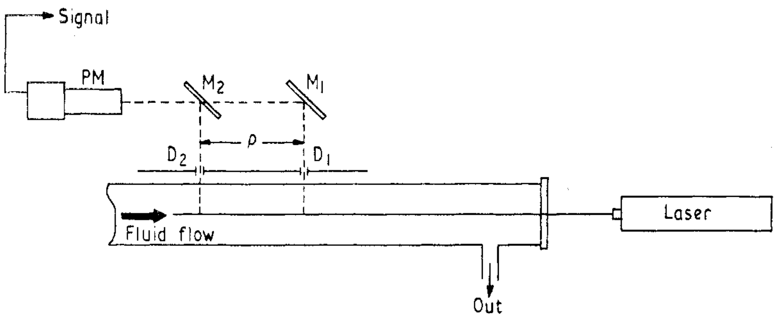

Figure 1. Experimental set-up. D_1, D_2 diaphragms; M_1 total reflecting mirror; M_2 half-silvered mirror; PM photomultiplier.

$V \ll V_c$ (where V_c is the typical correlation volume of the turbulent field). One can of course collect the light diffused at different angles by varying the position of the mirrors with respect to the axis of the tube. When the two mirrors are placed at a distance ρ, we have, with equal probability, $\boldsymbol{r}_{i,0}-\boldsymbol{r}_{j,0} = 0$, or $\boldsymbol{r}_{i,0}-\boldsymbol{r}_{j,0} = \boldsymbol{\rho}$, all other possibilities being excluded. Furthermore, as we shall see, we are interested in time intervals $\tau \ll t_c$ (where t_c is the typical correlation time of the turbulent field), so that $R_L(t''-t') \simeq 1$ and $R_L(t''-t'; \boldsymbol{r}_{i,0}-\boldsymbol{r}_{j,0}) \simeq R(\rho)$, the symbol R standing for the equal-time spatial correlation function

$$R(\rho) = \frac{\langle U_1(\boldsymbol{r}) U_1(\boldsymbol{r}+\boldsymbol{\rho}) \rangle}{\overline{U_1{}^2}}. \tag{3}$$

It is now easy to apply equation (2) to our case, obtaining

$$p_c(\rho, \tau) = \alpha \langle I \rangle \left(1 + \exp(-k^2 \overline{U_1{}^2}\tau^2) \frac{\exp(k^2 \overline{U_1{}^2}\tau^2) + \exp(k^2 \overline{U_1{}^2} R(\rho)\tau^2)}{2}\right)$$

$$= \alpha \langle I \rangle \left(\tfrac{3}{2} + \frac{\exp\{-k^2 U_1{}^2(1-R(\rho))\tau^2\}}{2}\right). \tag{4}$$

It can be more convenient, from an experimental point of view, to measure the correlation function of intensity fluctuation (Di Porto *et al.* 1969):

$$\langle \Delta I(t)\, \Delta I(0) \rangle = \langle I \rangle^2 \left[\exp\left\{-k^2 \overline{U_1{}^2} \int_0^\tau\!\!\int_0^\tau R_L(t''-t')\, dt''\, dt'\right\}\right.$$

$$\left. \times \left\langle \exp\left[k^2 \overline{U_1{}^2} \int_0^\tau\!\!\int_0^\tau \tilde{R}_L(t''-t'; \boldsymbol{r}_{i,0}-\boldsymbol{r}_{j,0})\, dt'\, dt''\right]\right\rangle_P \right]$$

$$= \langle I \rangle^2 \left(\tfrac{1}{2} + \frac{\exp\{-k^2 \overline{U_1{}^2}(1-R(\rho))\tau^2\}}{2}\right). \tag{5}$$

It is worthwhile to observe that the time dependent part of equations (4) and (5) has a Gaussian behaviour and its width $\Delta(\rho)$ is proportional to the quantity $(1-R(\rho))^{-1/2}$. Therefore we arrive at the remarkably simple relation

$$R(\rho) = 1 - \frac{\Delta^2(\infty)}{\Delta^2(\rho)} \tag{6}$$

where $\Delta(\infty)$ stands for $\Delta(\rho \gg V_c^{1/3})$ and use has been made of the property $R(\infty) = 0$. Typical values $k = 2^{1/2} \times 10^5\ \mathrm{cm}^{-1}$, $(\overline{U_1^2})^{1/2} \simeq 10^{-1}\ \mathrm{cm\ s}^{-1}$ (see e.g. Ling and Huang 1970) furnish a decay time $t_d \simeq 10^{-4}$ s for the time dependent part of equations (4) and (5), thus justifying the use of $R(\rho)$ in equation (4).

An interesting feature of the diffused light following from equation (4) is represented by the relation $p_c(\rho, \infty)/p_c(\rho, 0) = \frac{3}{4}$, which retains its validity also when $\rho \gg V_c^{1/3}$. In other words the Gaussian factorization property

$$\langle |E(t)\, E(0)|^2 \rangle = \langle |E(0)|^2 \rangle^2 + |\langle E(t)\, E(0) \rangle|^2 \tag{7}$$

is never verified, since it would imply $p_c(\rho, \infty)/p_c(\rho, 0) = \frac{1}{2}$ (Mandel and Wolf 1965). This peculiar statistical behaviour is an effect of couples of strongly correlated particles, which can never be neglected in our experimental situation.

We note that the modification undergone by equations (4) and (5), when the two scattering volumes cannot be considered equal, does not affect the validity of equation (6). In effect, if V_1, V_2 are the two scattering volumes, one has

$$p(0) = \frac{V_1^2 + V_2^2}{(V_1 + V_2)^2} \tag{8}$$

$$p(\rho) = \frac{2V_1 V_2}{(V_1 + V_2)^2} \tag{9}$$

where $p(0)$, $p(\rho)$ indicate the probabilities that two particles are in the same volume or not. It follows immediately that one can generalize equation (4) into

$$p_c(\rho, \tau) = \alpha \langle I \rangle [1 + p(0) + p(\rho) \exp\{-k^2 U_1^2 (1 - R(\rho))\tau^2\}] \tag{10}$$

which still yields equation (6) and implies $p_c(\rho, \infty)/p_c(\rho, 0) \geqslant \frac{3}{4}$, owing to the relation $p(0) \geqslant p(\rho)$.

We conclude by observing that, in many actual cases, the time $\bar{t}$ spent by a particle in crossing V_1 or V_2 is of some 10^{-2} s, so that $\bar{t} \gg t_d$, and no correction is needed for the finite transit time.

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